

Quantum mechanics emerges from information theory applied to causal horizons

Jae-Weon Lee*

*Department of energy resources development, Jungwon University,
5 dongburi, Goesan-eup, Goesan-gun Chungbuk Korea 367-805*

(Dated: March 1, 2011)

It is suggested that quantum mechanics is not fundamental but emerges from classical information theory applied to causal horizons. The path integral quantization and quantum randomness can be derived by considering information loss of fields or particles crossing Rindler horizons for accelerating observers. This implies that information is one of the fundamental roots of all physical phenomena. The connection between this theory and Verlinde's entropic gravity theory is also investigated.

PACS numbers: 03.65.Ta, 03.67.-a, 04.50.Kd

I. INTRODUCTION

Quantum mechanics and relativity are two pillars of modern physics. Recent developments of quantum information science revealed that quantum mechanics and relativity miraculously cooperate so as not to violate each other. For example, one can obtain a quantum mechanical state discrimination bound (Helstrom bound) from the no-signaling condition of the special relativity [1]. This connection is very surprising, because a quantum mechanical phenomenon is stochastic while relativity is deterministic. Furthermore, the holographic principle [2] or AdS/CFT correspondence [3] also asserts an unexpected connection between classical gravity in bulk and quantum mechanics on its surface. The origin of this mysterious connection is also still unknown.

Although quantum mechanics is experimentally and mathematically well established, its origin is not identified. For example, we do not know the origin of quantum randomness which leads to many paradoxes of quantum mechanics such as EPR paradox. This situation gave rise to numerous interpretations of quantum mechanics from the Copenhagen interpretation to the many worlds interpretation. One of the viable interpretations is information theoretic interpretation [4, 5], in which information about physical events plays a central role. For example, Zeilinger and Brukner [4, 5] introduced a notion of information space to describe quantum phenomena and suggested that quantum randomness arises from the discreteness of information (i.e., bits).

Why does physics have something to do with information? There are several hints for this question. Landauer's principle in quantum information science states that to erase information of a system irreversibly, energy should be consumed [6]. This means information is physical. In a series of works [7–11], based on this principle and quantum entanglement theory, the author and colleagues suggested that information is the key to understand the origins of dark energy [7], black hole mass [8] and even Einstein gravity [10, 11]. For example, it is sug-

gested that a cosmic causal horizon with a radius R_h has temperature $T_h \sim 1/R_h \sim H$, quantum informational entropy $S_h \sim R_h^2$ and hence a kind of thermal energy $E_h \sim T_h S_h \sim M_P^2 R_h \sim M_P^2/H$ which is comparable to dark energy observed [7]. Here, M_P is the reduced Planck mass and H is Hubble parameter. On the other hand, Jacobson [12] wrote a seminar paper linking the Einstein gravity to thermodynamics at Rindler horizons using the first law of thermodynamics $dE_h = T_h dS_h$. Recently, Verlinde [13] (See also [14]) suggested fascinating ideas linking gravitational force to entropic force and derived Newton's second law and the Einstein equation using similar horizon energy. This brought many related studies [15–38]. In [39] Lee suggested that Jacobson's [12] gravity theory or the quantum informational [10] theory can explain Verlinde's formalism of entropic gravity by identifying the holographic screen to be Rindler horizons for accelerating observers. (See also [40]).

Considering all these recent developments, it is plausible that quantum mechanics and gravity has information as a common ingredient, and information is the key to explain the strange connection between two. If gravity and Newton mechanics can be derived by considering information at Rindler horizons, it is natural to think quantum mechanics might have a similar origin. In this paper, along this line, it is suggested that quantum field theory (QFT) and quantum mechanics can be obtained from information theory applied to causal (Rindler) horizons, and that quantum randomness arises from information blocking by the horizons.

In Sec. II the connection between QFT and information theory is suggested. In Sec. III the connection between our theory and Verlinde's theory is investigated. Section IV contains discussions.

II. QUANTUM FIELD THEORY FROM INFORMATION THEORY

Basic assumptions in this paper are followings. First, we assume that the speed of information propagation (i.e., the light velocity c) is finite and, hence, there are causal horizons. Second, information is a fundamental ingredient of physics. That is, physics should reflect in-

* scikid@gmail.com

formation possessed by observers. Third, we assume the general principle of relativity (*not* general relativity of Einstein) stating that all observers are equivalent with respect to the formulation of the fundamental laws of physics [41]. Finally, we also assume some notions of classical information theory, spacetime, and its coordinate transformations.

Let us begin by considering an accelerating observer Θ_R with acceleration a in x_1 direction in a flat spacetime with coordinates $X = (t, x_1, x_2, x_3)$ (See Fig. 1). The Rindler coordinates $\xi = (\eta, r, x_2, x_3)$ for the observer are defined with

$$t = r \sinh(a\eta), \quad x_1 = r \cosh(a\eta) \quad (1)$$

on the Rindler wedges. There is an inertial observer Θ_M too. Now, consider a field ϕ crossing the Rindler horizon at a point P (actually, a two dimensional surface in $x_2 - x_3$ direction) and entering the future wedge F . A configuration for $\phi(x, t)$ is not necessarily meant to be classical but to be a function of spacetime carrying information.

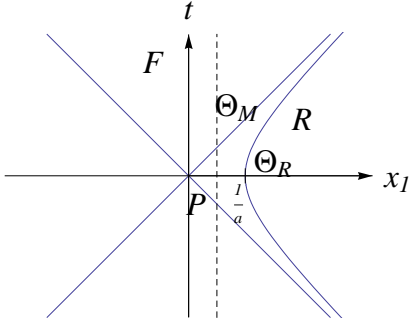


FIG. 1. Rindler chart for the observer Θ_R (curved line), who has no accessible information about field ϕ in a causally disconnected region F . The observer can only estimate a probabilistic distribution of the field, which turns out to be equal to that of a quantum field for inertial observer Θ_M (dashed line) in Minkowski spacetime.

As the field enters the Rindler horizon for the observer Θ_R , the observer shall have no more information about future configurations of ϕ and all what the observer can expect about ϕ evolution is a probabilistic distribution $P[\phi]$ of ϕ beyond the horizon. Already known information about ϕ becomes constraints for the distribution. I suggest that this ignorance is the origin of quantum randomness. According to our assumptions, information is fundamental in this theory, and what determines the physics in the wedge F is not a deterministic classical physics but the evolution of information itself. Thus, in this conjecture, the physics in the wedge should reflect the ignorance of the observer Θ_R about the field configurations, and there should not be a priori deterministic classical value for ϕ . That means for the observers there is no ‘objective physical reality’ such as classical fields before measurements. This is the main conjecture in this

paper, which naturally follows if we accept our assumptions.

This situation can be mathematically analyzed with classical information theory. The maximum ignorance about the field can be expressed by maximizing the Shannon information entropy $h[P[\phi]]$ of the possible (discrete) configurations $\Phi = \{\phi_i(X)\}$, $i = 1 \cdots n$ that the field may take beyond the horizon with probability $P[\phi_i]$. A uniform probability distribution may be adequate when there is no information about the events represented by random variables Φ . However, if there is a priori information available represented by l testable expectations (not a quantum expectation yet)

$$\langle f_k \rangle \equiv \sum_{i=1}^n P[\phi_i] f_k[\phi_i], \quad (k = 1 \cdots l), \quad (2)$$

we should use the principle of maximum entropy by Boltzmann to calculate the probability distribution $P[\Phi]$. Here, f_k , ($k = 1 \cdots l$) is a function of Φ and $\langle f_k \rangle$ is its expectation value with respect to $P[\Phi]$. According to the theorem, by maximizing the Shannon entropy

$$h[P] = - \sum_{i=1}^n P[\phi_i] \ln P[\phi_i], \quad (3)$$

with the constraints in Eq. (2) one can obtain the following probability distribution

$$P[\phi_i] = \frac{1}{Z} \exp \left[- \sum_{j=1}^l \lambda_j f_j(\phi_i) \right] \quad (4)$$

with a normalization constant (partition function) $Z = \sum_{i=1}^n \exp \left[- \sum_{j=1}^l \lambda_j f_j(\phi_i) \right]$, where λ_j ’s are Lagrangian multipliers satisfying the following relation $\langle f_k \rangle = - \frac{\partial}{\partial \lambda_k} \ln Z$. Thus, the usual Maxwell-Boltzmann distribution is a natural consequence of classical information theory, when there is information loss with constraints. Lisi [42] suggested a related derivation of the partition function by assuming a ‘universal action reservoir’.

What constraints can we impose on the motion of the field crossing the horizon? One constraint may come from the energy conservation

$$\sum_{i=1}^n P[\phi_i] H(\phi_i) = E, \quad (5)$$

where $H(\phi_i)$ is the Hamiltonian as a function of the field ϕ_i and E is its expectation. This comes from the fact that the energy expectation value of the field should not change. Another trivial one is the unity of the probabilities $\sum_{i=1}^n P[\phi_i] = 1$. Then, from the above theorem, the probability estimated by the Rindler observer, subject to these constraints, should be

$$P[\phi_i] = \frac{1}{Z} \exp [-\beta H(\phi_i)], \quad (6)$$

where β is the Lagrangian multiplier. Here, the partition function is

$$Z = \sum_{i=1}^n \exp[-\beta H(\phi_i)] = \text{tr } e^{-\beta H}, \quad (7)$$

and the trace is assumed to be performed with a (classical) discrete vector basis. Below, we shall take a continuum limit. It is important to recall that the field ϕ can not have a specific value before measurements according to our assumptions. The classical field theory could be obtained through extremization of $P[\phi_i]$ after establishing a QFT later. What is assumed here is that for both of the observers Θ_M and Θ_R , ϕ could have arbitrary values before measurements. Z represents this uncertainty.

As an example, let us consider a scalar field with Hamiltonian

$$H(\phi) = \int d^3x \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial t} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right] \quad (8)$$

with potential V . H alone, without a guiding principle, does not fully give us dynamics, neither classical nor quantum. For the Rindler observer with the coordinates (η, r, x_2, x_3) the proper time variance is $ard\eta$ and hence the Hamiltonian is changed to

$$H_R = \int_{r>0} dr dx_{\perp} ar \left[\frac{1}{2} \left(\frac{\partial \phi}{ar \partial \eta} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi}{\partial r} \right)^2 + \frac{1}{2} (\nabla_{\perp} \phi)^2 + V(\phi) \right], \quad (9)$$

where \perp denotes the plane orthogonal to (η, r) plane. Then, Eq. (7) becomes Eq. (2.5) of Ref. [43];

$$Z_R = \text{tr } e^{-\beta H_R}. \quad (10)$$

It is important to notice that Z (and hence Z_R) here is not a quantum partition function but a statistical partition function yet. Z_R simply represents the statistical system corresponding to the uncertain field configurations beyond the horizon.

Now, we need to show Z_R is equivalent to a quantum partition function. Fortunately, this is already done in Ref. [43]. Based on QFT, in the reference, it was shown that the real-time thermal Green's functions of the Rindler observer with Z_R are equivalent to the vacuum Green's function in Minkowski coordinates. (See [44] for a review.) Thus, as well known, the Minkowski vacuum is equivalent to thermal states for the Rindler observers. What I have newly shown here is that the thermal partition function Z_R assumed in Ref. [44] is from information loss about field configurations beyond the Rindler horizon and, therefore, the QFT formalism is equivalent to the purely information theoretic formalism suggested in this paper. Recall that Eq. (10) was derived without using any quantum physics in this paper. Since quantum mechanics can be thought to be single particle limit of QFT, this implies also that quantum mechanics

emerges from information theory applied to Rindler horizons and is not fundamental. Although the reasoning is simple, conclusions can be far-reaching.

To be concrete, it is worthy to briefly repeat the calculation in Ref. [44]. There, it was shown by analytical continuation that in the Rindler coordinates Z_R is *mathematically* equivalent to

$$Z_R = N_0 \int_{\phi(0)=\phi(\beta')} D\phi \exp \left\{ -\alpha \int_0^{\beta'} d\tilde{\eta} \int_{r>0} dr dx_{\perp} ar \left[\frac{1}{2} \left(\frac{\partial \phi}{ar \partial \tilde{\eta}} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi}{\partial r} \right)^2 + \frac{1}{2} (\nabla_{\perp} \phi)^2 + V(\phi) \right] \right\}, \quad (11)$$

where we explicitly denoted a constant α having a dimension of $1/H_R t$ for a dimensional reason. Thus, $\beta = \alpha \beta'$.

The trace turned into the periodic boundary condition $\phi(\tilde{\eta} = 0) = \phi(\tilde{\eta} = \beta')$ as usual. By further changing integration variables as $\tilde{r} = r \cos(a\tilde{\eta})$, $\tilde{t} = r \sin(a\tilde{\eta})$ and choosing $\beta' = 2\pi/a \equiv 1/\alpha k_B T_U$ the region of integration is transformed from $0 \leq \tilde{\eta} \leq \beta'$, $0 \leq r \leq \infty$ into the full two dimensional flat space $-\infty \leq \tilde{t} \leq \infty$, $-\infty \leq \tilde{r} \leq \infty$. Of course, this specific β' value leads to Unruh temperature $T_U = a/2\alpha\pi k_B$, where k_B is the Boltzman constant. From the well-known QFT result, one can find $1/\alpha$ to be \hbar . Since $\hbar = 1/\alpha$ is from the lagrange multiplier β , the Planck constant \hbar is associated with the change of Z_R by energy change, that is, \hbar is some fundamental temperature given by nature.

Then, the partition function becomes

$$Z_Q^E = N_1 \int D\phi \exp \left\{ -\alpha \int d\tilde{r} d\tilde{t} dx_{\perp} \left[\frac{1}{2} \left(\frac{\partial \phi}{\partial \tilde{t}} \right)^2 + \frac{1}{2} \left(\frac{\partial \phi}{\partial \tilde{r}} \right)^2 + \frac{1}{2} (\nabla_{\perp} \phi)^2 + V(\phi) \right] \right\} \quad (12)$$

$$= N_1 \int D\phi \exp \left\{ -\frac{I_E}{\hbar} \right\}.$$

where I_E is the Euclidean action for the scalar field in the inertial frame. By analytic continuation $\tilde{t} \rightarrow it$, one can see Z_Q^E becomes the usual zero temperature quantum mechanical partition function Z_Q for ϕ . Since both of Z_R and Z_Q can be obtained from Z_Q^E by analytic continuation, they are physically equivalent as pointed out in Ref. [44]. A partition function contains all information about a statistical system. Thus, it is enough to show the equivalence of two partition functions to prove the equivalence of QFT and the information theoretic model suggested in this paper, once we accept the information theoretic origin of Z_R . Of course, one can reverse the logic and obtain Z in Eq. (10) from Z_Q . Now we see that quantum fluctuations correspond to the ignorance of Rindler observers about the fields beyond Rindler horizons.

III. QUANTUM MECHANICS AND ENTROPIC GRAVITY

It is straightforward to extend the previous analysis to quantum mechanics for point particles. We can imagine a point particle at a point P just crossing the Rindler horizon and entering the future wedge F . Like the case of the field, the Rindler observer gets no more information from the particle. This maximal ignorance is represented by probability distribution $P[x_i(t)]$ for the i -th possible path that the particle may take.

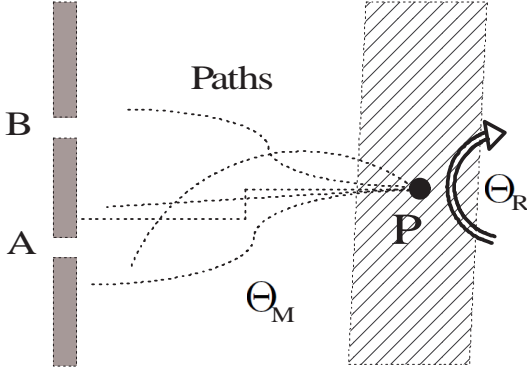


FIG. 2. The Rindler observer Θ_R has no more information about paths of the particle crossing the horizon (shaded region) and all what the observer can expect about the particle is a probabilistic distribution of its motion. This seems to be the origin of quantum randomness of the motion. Here, A and B represent slits for a typical double-slit experiment.

Then, the partition function is

$$Z_R = \sum_{i=1}^n \exp[-\beta H(x_i)] = \text{tr } e^{-\beta H}, \quad (13)$$

where H is the point particle Hamiltonian now. Since the usual point particle quantum mechanics is a non-relativistic and single particle limit of the quantum field theory, we expect Z_R is equal to the quantum partition function for the particle with mass m in Minkowski space-time

$$\begin{aligned} Z_Q &= \\ N_2 \int Dx \exp \left[-\frac{i}{\hbar} \int d\tilde{t} \left\{ \frac{m}{2} \left(\frac{\partial x}{\partial \tilde{t}} \right)^2 - V(x) \right\} \right] \\ &= N_1 \int Dx \exp \left\{ -\frac{i}{\hbar} I(x_i) \right\}, \end{aligned} \quad (14)$$

where I is the action. Then, as is well known one can associate each path x_i with a wave function $\psi \sim e^{-iI}$, which leads to Schrödinger equation for ψ [45].

This interpretation could shed a new light on the paradoxical behaviors of quantum particles. For example, consider the double slit experiment. the Rindler observer Θ_R , having no access to the information about paths of the particle, could not say which path of two slits (A

or B) in Fig. 2 was chosen by the particle. Otherwise, it will violate the no-signaling principle. According to our conjecture, physics in the wedge F should reflect this ignorance. Thus, the particle could not have a deterministic path before measurement. On the other hand, the observer Θ_M who can measure the paths, after he or she enters the horizon (For this observer light cones play a role of causal horizons.), has a chance to know the “which-way” information. This could induce the “wave function collapse”. According to our theory, wave functions or states are neither particles nor physical waves but just probability functions about information, thus there is no need for a concern regarding immediate superluminal changes of wavefunctions.

This theory also gives some new insights on the origin of Verlinde’s entropic gravity theory. In his papers identities of information and its entropy are not so clearly given. Therefore, there are several concerns [25, 40, 46–48] on the assumptions Verlinde took. Two important concerns are about the origin of the entropy variation formula (Eq. (15) below) and identity of the holographic screen.

According to our theory the entropy which is associated with the entropic force is the entropy $h[P(x_i)]$ about unobservable paths, estimated by the Rindler observers. Culetu [25] pointed out that if the screen plays the role of a local Rindler horizon at $r = c^2/a$, Verlinde’s entropy formula can be explained. This interpretation is in accordance with Lee et al’s proposal [10, 39] that the Einstein equation represents information erasing process at Rindler horizons.

The results in this paper enhance these interpretations. For the observer at $r = c^2/a$, the Rindler Hamiltonian becomes a physical Hamiltonian generating η translation [44]. Thus, this distance r is special. The observer shall have no more path information of the non-relativistic particle with mass m crossing the horizon. In this case the loss of information of the particle results in the horizon entropy S_h increase [39] as

$$\Delta S_h = \frac{\Delta E_h}{T_U} = \frac{2\pi\kappa_B m r}{\hbar}, \quad (15)$$

which is just the entropy variation Verlinde assumed. Here, we used the Unruh temperature and the horizon energy variation $\Delta E_h \simeq mc^2$ due to the holographic principle. Once we obtain this formula, it is straightforward to reproduce Newton’s equation and gravity in Verlinde’s formalism with the equipartition energy law.

The maximum entropy proposal in Verlinde’s theory can be also understood in this way. From Eq. (13) free energy can be expressed as

$$F = -\frac{1}{\beta} \ln Z_R. \quad (16)$$

The classical path corresponds to the saddle point approximation ($Z_R \sim \exp[-\beta I_E(x_{cl})]$) [49]

$$F \simeq F_{cl} = -\frac{1}{\beta} (-\beta I_E(x_{cl})) = I_E(x_{cl}), \quad (17)$$

where $I_E(x_{cl})$ is the Euclidean action for classical path satisfying the Lagrange equation. Since the maximum entropy is achieved when F is minimized, we see that classical physics with the minimum action corresponds to a maximum entropy (of the paths) condition, that is, the classical path is the typical path maximizing the entropy $h[P]$ with the constraints for the Rindler observer. The holographic principle demands that the increase of the horizon entropy should be equal to the entropy of the paths of the particle entered.

In short, Verlinde's holographic screen is just Rindler horizons and its entropy is associated with the lost path information of the particle crossing the horizons [39]. Then, there is an entropic force linked to this information loss which can be calculated by using the first law or Landauer's principle. Thus, our theory provides a natural model for the entropy and information which Verlinde assumed.

IV. DISCUSSIONS

In this paper it is shown that if there is a causal boundary for an observer, the observer could expect statistical distribution for physical objects beyond the horizon due to information loss. For another observer who can access the objects this thermal distribution corresponds to just quantum fluctuation.

What are the merits of our new interpretation of quantum physics? First, this theory explains the strange connection between quantum mechanics and special relativ-

ity such as no-signaling condition in quantum measurements. Since our formalism of quantum mechanics itself emerges from the limitation of the information propagation velocity, it is natural that we can not send a classical superluminal signal even with quantum nonlocal correlation (entanglement) by any means. Second, from this fact, it might give us a new hint to study of unification of gravity and quantum mechanics. Third, this model could also explain the origin of Verlinde's formalism about Newton mechanics and gravity. Our theory is in accordance with the quantum informational dark energy model [7] too.

In summary, it is shown that the path integral quantization and quantum randomness can be derived by considering information loss behind Rindler horizons. Quantum mechanics is not fundamental and emerges from information theory accompanied with the Rindler coordinate transformation. This implies that quantum mechanics is more about information rather than particles or waves. Thus, now we have some striking relationships among information, gravity, Newtonian mechanics, and even quantum mechanics. Information seems to be one of the roots of all physical phenomena.

ACKNOWLEDGMENTS

This work was supported in part by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the ministry of Education, Science and Technology (2010-0024761) and the topical research program (2010-T-1) of Asia Pacific Center for Theoretical Physics.

-
- [1] J. Bae, J.-W. Lee, J. Kim, and W.-Y. Hwang, Phys. Rev. A **78**, 022335 (2008).
 - [2] G. 't Hooft, *Salam-festschrift* (World Scientific, Singapore, 1993).
 - [3] O. Aharony, S. S. Gubser, J. Maldacena, H. Ooguri, and Y. Oz, Phys. Rep. **323**, 183 (2000).
 - [4] A. Zeilinger, Found. Phys. **29**, 631 (1999).
 - [5] C. Brukner and A. Zeilinger, Physical Review Letters **83**, 3354 (1999).
 - [6] R. Landauer, IBM J. Res. Dev. **5**, 183 (1961).
 - [7] J.-W. Lee, J. Lee, and H.-C. Kim, JCAP08(2007)005 (2007), hep-th/0701199.
 - [8] H.-C. Kim, J.-W. Lee, and J. Lee, Mod. Phys. Lett. A **25**, 1581 (2007), 0709.3573.
 - [9] H.-C. Kim, J.-W. Lee, and J. Lee, JCAP **0808**, 035 (2008), 0804.2579.
 - [10] J.-W. Lee, H.-C. Kim, and J. Lee, (2010), 1001.5445.
 - [11] J.-W. Lee, *Entanglement and Gravity*, Talk given at the KIAS Workshop on Quantum Information Sciences Aug. 19, 2009, Seoul Korea (2009).
 - [12] T. Jacobson, Phys. Rev. Lett. **75**, 1260 (1995).
 - [13] E. P. Verlinde, arXiv:1001.0785 (2010).
 - [14] T. Padmanabhan, arXiv:0912.3165 (2009).
 - [15] L. Zhao, (2010), 1002.0488.
 - [16] R.-G. Cai, L.-M. Cao, and N. Ohta, (2010), 1002.1136.
 - [17] R.-G. Cai, L.-M. Cao, and N. Ohta, (2010), 1001.3470.
 - [18] Y. S. Myung, (2010), 1002.0871.
 - [19] Y.-X. Liu, Y.-Q. Wang, and S.-W. Wei, arXiv:1002.1062 (2010).
 - [20] Y. Tian and X. Wu, (2010), 1002.1275.
 - [21] A. Pesci, (2010), 1002.1257.
 - [22] M. Diego, (2010), arXiv:1002.1941.
 - [23] I. V. Vanea and M. A. Santos, (2010), arXiv:1002.2454.
 - [24] R. A. Konoplya, (2010), 1002.2818.
 - [25] H. Culetu, (2010), arXiv:1002.3876.
 - [26] Y. Zhao, (2010), arXiv:1002.4039.
 - [27] S. Ghosh, (2010), arXiv:1003.0285.
 - [28] J. Munkhammar, (2010), arXiv:1003.1262.
 - [29] X. Kuang, Y. Ling, and H. Zhang, (2010), arXiv:1003.0195.
 - [30] H. Wei, (2010), 1005.1445.
 - [31] J. R. Mureika and R. B. Mann, (2010), 1005.2214.
 - [32] Y.-F. Cai, J. Liu, and H. Li, (2010), 1003.4526.
 - [33] A. Morozov, (2010), 1003.4276.
 - [34] R. Casadio and A. Gruppiso, (2010), 1005.0790.
 - [35] T. Padmanabhan, (2010), 1003.5665.
 - [36] R. Horvat, (2010), 1003.4363.
 - [37] Q. Pan and B. Wang, (2010), 1004.2954.
 - [38] L. Modesto and A. Randono, (2010), 1003.1998.
 - [39] J.-W. Lee, (2010), 1003.4464.

- [40] H. Culetu, (2010), 1005.1570.
- [41] C. Møller, *The theory of relativity* (Oxford University Press, Delhi, 1974).
- [42] A. G. Lisi, arXiv:physics/0605068 (2006).
- [43] W. G. Unruh and N. Weiss, Phys. Rev. D**29**, 1656 (1984).
- [44] L. C. B. Crispino, A. Higuchi, and G. E. A. Matsas, Rev. Mod. Phys. **80**, 787 (2008), 0710.5373.
- [45] D. Derbes, American Journal of Physics **64**, 881 (1996).
- [46] M. Li and Y. Wang, arXiv:1001.4466 (2010).
- [47] S. Gao, (2010), arXiv:1002.2668.
- [48] J.-P. Lee, (2010), 1005.1347.
- [49] R. Banerjee and B. R. Majhi, (2010), 1003.2312.